200C 03.6.

Tribology

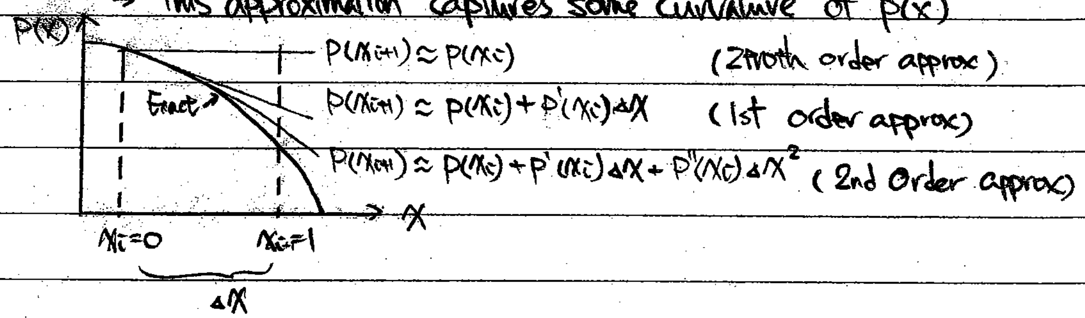
* Finit Difference Method.

The Taylor series provide a means to predict a function's value at one part in terms of the function value and its derivatives at another point.

(1) $P(X_{i+1}) \approx P(X_i) + P'(X_i)(X_{i+1} - X_i)$ (1st order approx)

(2) $P(X_{i+1}) \approx P(X_i) + P'(X_i)(X_{i+1} - X_i) + \frac{1}{2!} P''(X_i)(X_{i+1} - X_i)^2$ (2nd order approx)

↳ This approximation captures some curvature of $p(x)$



So, using Taylor series, we can develop a numerical 1st derivative

$$P(X_{i+1}) = P(X_i) + P'(X_i)(X_{i+1} - X_i) + \frac{1}{2!} P''(X_i)(X_{i+1} - X_i)^2 + \cdots + R_N$$

Using the Nth order approximation form from Eq. 1, we have

$$P'(X_i) = \underbrace{\frac{P(X_{i+1}) - P(X_i)}{X_{i+1} - X_i}}_{\text{Divided Difference Formula for } P'(X_i)} - \frac{R_i}{X_{i+1} - X_i} \quad \text{where } \Delta X = X_{i+1} - X_i$$

* Central Finite - Divided Difference

1st Derivative

$$* \quad P'(X_i) = \frac{P(X_{i+1}) - P(X_{i-1})}{2\Delta X}$$

$$P'(X_i) = \frac{-P(X_{i+2}) + 8P(X_{i+1}) - 8P(X_{i-1}) + P(X_{i-2})}{12\Delta X}$$

2nd Derivative

$$* \quad P''(X_i) = \frac{P(X_{i+1}) - 2P(X_i) + P(X_{i-1})}{\Delta X^2}$$

Backward Finite-Divided Difference

$$p'(x_i) = \frac{P(x_i) - P(x_{i-1})}{\Delta x}$$

$$p''(x_i) = \frac{P(x_i) - 2P(x_{i-1}) + P(x_{i-2})}{\Delta x^2}$$

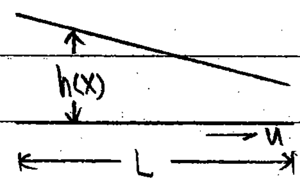
Forward Finite-Divided Difference

$$p'(x_i) = \frac{P(x_{i+1}) - P(x_i)}{\Delta x}$$

$$p''(x_i) = \frac{P(x_{i+2}) - 2P(x_{i+1}) + P(x_i)}{\Delta x^2}$$

* Numerical Solution to Reynolds Equation

For planar slider bearings.



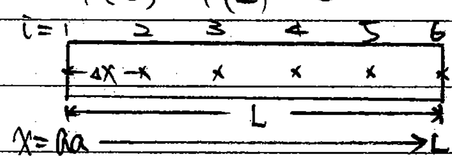
$$h(x) = H\left(1 - \alpha\frac{x}{L}\right)$$

Let $\alpha = 0.5$

1D Reynold's Equation is

$$\frac{d}{dx}\left(h^3 \frac{dp}{dx}\right) = 6\eta u \frac{dh}{dx}$$

$$P(0) = P(L) = 0$$



$x = aa + (i-1)\Delta x \Rightarrow x_i$ for $i = 1 \cdots N_x$

$P(x) \rightarrow P(x_i) \Rightarrow P_i$ for $i = 1 \cdots N_x$

$h(x) \rightarrow h(x_i) \Rightarrow h_i$ for $i = 1 \cdots N_x$

Write Reynolds ODE in linear form $[\, A(x)\,p'' + B(x)\,p' = g(x)\,]$

$$h^3 \frac{d^2p}{dx^2} + 3h^2 \frac{dp}{dx}\frac{dh}{dx} = 6\eta u \frac{dh}{dx}$$ Now, discretize.

Spring Break : 3/...

24 100C
0318.

Tribology

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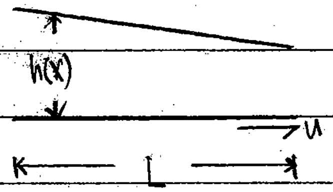
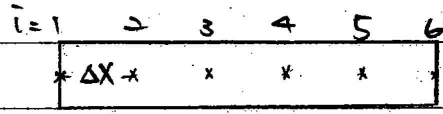


$$h(x) = H\left(1-\alpha\frac{x}{L}\right)$$

Reynolds Equation

$$h^3\frac{d^2P}{dx^2} + 3h^2\frac{dP}{dx}\frac{dh}{dx} = 6\eta u\frac{dh}{dx}.$$

Imagine that the pressure along the length L changes from boundary to boundary, we represent this change by dividing the length into discretized points. For simplicity, we'll do just 6 ($N_x = 6$)



$$\Delta x = \frac{bb-aa}{N_x} \text{ and } N_x = 6$$

$x = aa \longrightarrow bb$

$aa = 0$ & $bb = L$ For this problem

$$\frac{df}{dx} = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\begin{cases} x = aa + (i-1)\Delta x \Rightarrow x_i \\ P(x) \Rightarrow P(x_i) \Rightarrow P_i \\ h(x) \Rightarrow h(x_i) \Rightarrow h_i = H\left(1-\alpha\frac{x_i}{L}\right) \end{cases}$$

Write ODE in linear form $[A(x)P'' + B(x)P' = g(x)]$

$$\Rightarrow h^3\frac{d^2P}{dx^2} + 3h^2\frac{dP}{dx}\frac{dh}{dx} = 6\eta u\frac{dh}{dx} \quad *1. \text{ governing equation.}$$

$$\Rightarrow h_i^3\left(\frac{P_{i+1}-2P_i+P_{i-1}}{\Delta x^2}\right) + 3h_i^2\left(\frac{dh}{dx_i}\right)\left(\frac{P_{i+1}-P_{i-1}}{2\Delta x}\right) = 6\eta u\left(\frac{h_{i+1}-h_{i-1}}{2\Delta x}\right) \quad *2.$$

Boundary Equations:

$$\begin{cases} P(0) = P_i = 0 \text{ for } i=1 \\ P(L) = P_i = 0 \text{ for } i = N_x \end{cases}$$

↑ This is pressure written in numerical form.

From Equation #2.

$$\underbrace{P_{i-1}\left(\frac{h_i^3}{\Delta x^2}-\frac{3h_i^2}{2\Delta x}\frac{dh}{dx}\right)}_{aW_i}+\underbrace{P_i\left(\frac{-2h_i^3}{\Delta x^2}\right)}_{aP_i}+\underbrace{P_{i+1}\left(\frac{h_i^3}{\Delta x^2}+\frac{3h_i^2}{2\Delta x}\frac{dh}{dx}\right)}_{aE_i}=\underbrace{6\eta u\left(\frac{h_{i+1}-h_{i-1}}{2\Delta x}\right)}_{g_i}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ aW_2 & aP_2 & aE_2 & 0 & 0 & 0 \\ 0 & aW_3 & aP_3 & aE_3 & 0 & 0 \\ 0 & 0 & aW_4 & aP_4 & aE_4 & 0 \\ 0 & 0 & 0 & aW_5 & aP_5 & aE_5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\substack{\text{Tri-Diagonal Matrix} \\ a_{ij} \\ \text{Row} \quad \text{column}}}\underbrace{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix}}_{\substack{P_i \\ \text{vector}}}=\underbrace{\begin{bmatrix} 0 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ 0 \end{bmatrix}}_{\substack{b_i \\ \text{vector}}}$$

solve $A_{ij}\cdot P_i = g_i$

in Mathematica $\quad$ P= linearSolve[a, b]

in Matlab $\quad$ P= a / b